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THEORETICAL AND EXPERIMENTAL INVESTIGATION OF VIBRATION OF MULTILAYER PLATES UNDER THE ACTION OF IMPULSE AND IMPACT LOADS

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Abstract—A refined theory is used for investigating the strain–stressed state (SSS) of multilayer plates under impulse and impact loading. The theory takes into account the transverse shear strains in each of the layers. The broken line hypothesis holds true for a package. Experimental determination of the SSS is based on the dynamic wide-range strain measurement technique. The numerical results obtained are compared with experimental data.

INTRODUCTION

Multilayer structures are widely used in different branches of engineering, since they can combine properties which are impossible to obtain by using any one of the structural materials available. Thus, along with high strength, stiffness and load-carrying capacity, the appropriate levels of noise, vibrational and thermal insulation may be provided, as well as corrosion and radiation resistance.

Multilayer elements of structures comprising plates or shells combine, in the general case, layers of different thickness with various mechanical characteristics. In a number of cases such elements cannot be considered as thin ones. Therefore, the strain–stressed state (SSS) of such structures are described most authentically within the framework of the theory of elasticity. However this has considerable computational difficulties.

Another more simple method exists when the behaviour of a multilayer structure is described in terms of two-dimensional theory equations. Three-dimensional problems are reduced to two-dimensional ones by making different assumptions about the SSS character, depending on the relationship of the geometric and mechanical parameters of the layers of a specific shell or plate. The use of different combinations of one or other hypothesis accounts for a large variety of computational schemes. As noted in the reviews of Grigoliuk and Kogan (1972), Grigoliuk and Kulikov (1988), as well as in that of Reddy (1989), the theory of multilayer shells and plates is being developed along two main lines.

The first one is related to works in which the three-dimensional problem is reduced to a two-dimensional one on the basis of hypotheses applied to the entire package of layers as a whole. The simplest way of solving this problem is by using the Kirchhoff–Love and S. P. Timoshenko hypotheses.

The second, more general line, is related to works in which hypotheses for each separate layer are used for deriving the equations. In so doing, the system order depends on the number of layers.

The design of multilayer plates and shells subjected to the action of static loads, as well as the solution of the problem of their stability and natural oscillations is dealt with in a large number of publications. And as for the problems of non-stationary strain of multilayer elements in structures, they are insufficiently elucidated in literature, therefore the problem of design of such elements under the action of impulse and impact loads remains a vital one.

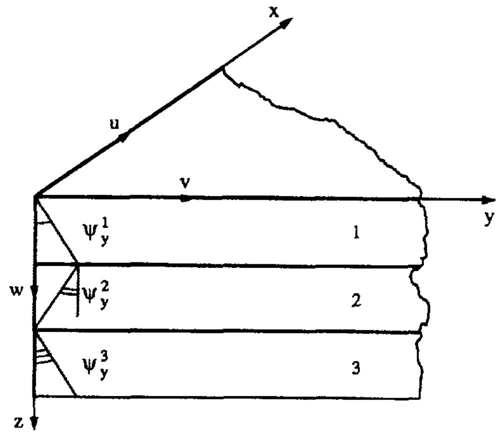

Fig. 1. Multilayer plate.

This work describes the method of investigating non-stationary vibrations of multilayer plates on the basis of the hypotheses of the refined theory of S. P. Timoshenko for each layer. Displacements and the external load are expanded into Fourier series for functions satisfying the problem boundary conditions.

In the case of impact loading, the contact approach of bodies is computed according to Hertz's theory of compression of contacting bodies.

Experimental investigations of the SSS of multilayer plates under impact loading by a spherical steel indentor are based on the dynamic wide-range strain measurement technique (Vorobiev *et al.*, 1989). The technique ensures detection of the current time-dependent strain values and measurement of time intervals.

The results of numeric and experimental investigations of the SSS of multilayer plates under impact by a steel ball are given, and the influence of the structural parameters on the SSS of plates under impulse loading is studied.

THEORETICAL METHOD OF INVESTIGATION

A multilayer freely supported plate with layers of constant thickness (Fig. 1) is selected for the computational scheme.

The strain in layers is described within the framework of the refined theory of plates, accounting for the transverse shear in each layer, with adoption of the broken normal line for the package as a whole. At this, the deflection is considered to be constant over the thickness, since it is assumed that the layer materials are non-compressible in the transverse direction. It is also assumed that the contact between the layers excludes their delamination and mutual slipping.

The kinematic hypotheses (Grigoliuk and Chulkov, 1964) accepted notes the displacements of a point in the ith layer in the direction of the coordinate axes x, y, z as follows:

$$u^i = u + \sum_{j=1}^{i-1} h_j \psi_x^j + (z - \delta_{i-1})\psi_x^i,$$

$$v^i = v + \sum_{j=1}^{i-1} h_j \psi_y^j + (z - \delta_{i-1})\psi_y^i,$$

$$w^i = w, \tag{1}$$

where

$$\delta_i = \sum_{j=1}^{i} h_j, \quad \delta_{i-1} \leqslant z \leqslant \delta_i\,; i = \overline{1, I}\,;$$

$u = u(x, y, t)$, $v = v(x, y, t)$, $w = w(x, y, t)$ are the displacements of a point in the coordinate plane in the direction of the coordinate axes; $\psi_x^i = \psi_x^i(x, y, t)$, $\psi_y^i = \psi_y^i(x, y, t)$ $(i = \overline{1, I})$ are the angles of rotation of the normal within the limits of the ith layer (t is the time).

The strains in the layers are determined from Cauchy's formulae:

$$\varepsilon_x^i = \mathrm{u}_{,x} + \sum_{j=1}^{i-1} h_j \psi_{x,x}^j + (z - \delta_{i-1})\psi_{x,x}^i,$$

$$\varepsilon_y^i = v_{,y} + \sum_{j=1}^{i-1} h_j \psi_{y,y}^j + (z - \delta_{i-1})\psi_{y,y}^i,$$

$$\gamma_{xy}^i = \gamma_{yx}^i = u_{,y} + v_{,x} + \sum_{j=1}^{i-1} h_j(\psi_{x,y}^j + \psi_{y,x}^j) + (z - \delta_{i-1})(\psi_{x,y}^i + \psi_{y,x}^i),$$

$$\gamma_{xz}^i = \gamma_{zx}^i = \psi_x^i + w_{,x}, \gamma_{yz}^i = \gamma_{zy}^i = \psi_y^i + w_{,y}, i = \overline{1, I}. \tag{2}$$

The relationship between the stresses and strains in the ith layer is specified by Hooke's law:

$$\sigma_x^i = \frac{E_i}{1 - v_i^2}(\varepsilon_x^i + v_i \varepsilon_y^i),$$

$$\sigma_y^i = \frac{E_i}{1 - v_i^2}(\varepsilon_y^i + v_i \varepsilon_x^i),$$

$$\tau_{xy}^i = \tau_{yx}^i = \frac{E_i}{2(1 + v_i)}\gamma_{xy}^i,$$

$$\tau_{xz}^i = \tau_{zx}^i = \frac{E_i}{2(1 + v_i)}\gamma_{xz}^i,$$

$$\tau_{yz}^i = \tau_{zy}^i = \frac{E_i}{2(1 + v_i)}\gamma_{yz}^i, i = \overline{1, I}, \tag{3}$$

where E_i is Young's modulus, v_i is Poisson's ratio of the ith layer.

The equations of motion of a multilayer plate subjected to a transverse load P are obtained with the help of Hamilton's variational principle (Shupikov *et al.*, 1992), and their structure is similar to that given in the work by Grigoliuk and Chulkov (1964) for the case of static loading:

$$C_1^I u_{,xx} + C_3^I u_{,yy} + (C_2^I + C_3^I)v_{,xy} + \sum_{i=1}^{I} [D_1^i \psi_{x,xx}^i + D_3^i \psi_{x,yy}^i + (D_2^i + D_3^i)\psi_{y,xy}^i] = 0\,;$$

$$(C_2^I + C_3^I)u_{,xy} + C_3^I v_{,xx} + C_1^I v_{,yy} + \sum_{i=1}^{I} [(D_2^i + D_3^i)\psi_{x,xy}^i + D_3^i \psi_{y,xx}^i + D_1^i \psi_{y,yy}^i] = 0\,;$$

$$C_3^I(w_{,xx} + w_{,yy}) + \sum_{i=1}^{I} \alpha_3^i(\psi_{x,x}^i + \psi_{y,y}^i) - C_\rho^I w_{,tt} + P = 0\,;$$

$$D_1^i u_{,xx} + D_3^i u_{,yy} + (D_2^i + D_3^i) v_{,xy} - \alpha_3^i (\psi_x^i + w_{,x})$$
$$+ \sum_{j=1}^{I} \left\{ \begin{bmatrix} D_1^i h_j, & j < i \\ K_1^i, & j = i \\ D_1^j h_i, & j > i \end{bmatrix} \psi_{x,xx}^j + \begin{bmatrix} D_3^i h_j, & j < i \\ K_3^i, & j = i \\ D_3^j h_i, & j > i \end{bmatrix} \psi_{x,yy}^j + \begin{bmatrix} (D_2^i + D_3^i) h_j, & j < i \\ K_2^i + K_3^i, & j = i \\ (D_2^j + D_3^j) h_i & j > i \end{bmatrix} \psi_{y,xy}^i \right\} = 0;$$

$$(D_2^i + D_3^i) u_{,xy} + D_3^i v_{,xx} + D_1^i v_{,yy} - \alpha_3^i (w_{,y} + \psi_y^i)$$
$$+ \sum_{j=1}^{I} \left\{ \begin{bmatrix} (D_2^i + D_3^i) h_j, & j < i \\ K_2^i + K_3^i, & j = i \\ (D_2^j + D_3^j) h_i & j > i \end{bmatrix} \psi_{x,xy}^j + \begin{bmatrix} D_3^i h_j, & j < i \\ K_3^i, & j = i \\ D_3^j h_i, & j > i \end{bmatrix} \psi_{y,xx}^j \right.$$
$$\left. + \begin{bmatrix} D_1^i h_j, & j < i \\ K_1^i, & j = i \\ D_1^j h_i, & j > i \end{bmatrix} \psi_{y,yy}^j \right\} = 0; \quad (4)$$

$$i = \overline{1, I}.$$

The following expressions were obtained for the boundary conditions corresponding to free support:

$$x = 0, \quad x = A;$$

$$C_1^I u_{,x} + C_2^I v_{,y} + \sum_{i=1}^{I} [D_1^i \psi_{x,x}^i + D_2^i \psi_{y,y}^i] = 0;$$

$$v = 0;$$

$$w = 0;$$

$$D_1^i u_{,x} + D_2^i v_{,y} + \sum_{j=1}^{I} \left\{ \begin{bmatrix} D_1^i h_j, & j < i \\ K_1^i, & j = i \\ D_1^j h_i, & j > i \end{bmatrix} \psi_{x,x}^j + \begin{bmatrix} D_2^i h_j, & j < i \\ K_2^i, & j = i \\ D_2^j h_i, & j > i \end{bmatrix} \psi_{y,y}^j \right\} = 0;$$

$$\psi_y^i = 0;$$

$$y = 0, \quad y = B;$$

$$u = 0;$$

$$C_2^I u_{,x} + C_1^I v_{,y} + \sum_{i=1}^{I} [D_2^i \psi_{x,x}^i + D_1^i \psi_{y,y}^i] = 0;$$

$$w = 0;$$

$$\psi_x^i = 0;$$

$$D_2^i u_{,x} + D_1^i v_{,y} + \sum_{j=1}^{I} \left\{ \begin{bmatrix} D_2^i h_j, & j < i \\ K_2^i, & j = i \\ D_2^j h_i, & j > i \end{bmatrix} \psi_{x,x}^j + \begin{bmatrix} D_1^i h_j, & j < i \\ K_1^i, & j = i \\ D_1^j h_i, & j > i \end{bmatrix} \psi_{y,y}^j \right\} = 0; \quad (5)$$

where

$$\alpha_1^i = \frac{E_i h_i}{1-\nu_i^2};\quad \alpha_2^i = \frac{E_i h_i \nu_i}{1-\nu_i^2};\quad \alpha_3^i = \frac{E_i h_i}{2(1+\nu_i)};$$

$$\beta_k^i = h_i \alpha_k^i;\quad \gamma_k^i = h_i \beta_k^i;\quad C_k^i = \sum_{j=1}^{i} \alpha_k^j;\quad C_\rho^i = \sum_{j=1}^{i} h_j \rho_j;$$

$$D_k^i = h_i (C_k^I - C_k^i) + \frac{\beta_k^i}{2};\quad K_k^i = h_i^2 (C_k^I - C_k^i) + \frac{\gamma_k^i}{3};\quad i = \overline{1, I},\quad k = 1, 2, 3.$$

Here ρ_i is the ith layer ($i = \overline{1, I}$) material density.

In case of impact loading, system (4) should be complemented by the equation of motion of a load, and by the equation of joint dynamic displacement of the load and plate in the point of contact. The equation of motion of a load with the mass M, and the initial conditions have the form

$$MZ_{,tt} = Mg - F,\quad Z(0) = 0,\quad Z_{,t}(0) = \sqrt{2gH}. \tag{6}$$

Here Z is the dynamic load displacement; g is the free-fall acceleration; H is the load falling height; F is the force of interaction of the load and plate in the place of contact

$$F = \iint_S P\,\mathrm{d}S,$$

S is the spot of contact interaction.

The contact force F is determined from the condition of joint displacement of the load Z and plate w with account of the contact approach

$$Z - w - kF^{2/3} = 0. \tag{7}$$

The system of equations (4) is solved jointly with eqns (6) and (7).

The displacements and external load are expanded into Fourier series for functions which satisfy the boundary conditions corresponding to free edge support:

$$u(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{1mn}(t) \cos\frac{m\pi x}{A} \sin\frac{n\pi y}{B};$$

$$v(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{2mn}(t) \sin\frac{m\pi x}{A} \cos\frac{n\pi y}{B};$$

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{3mn}(t) \sin\frac{m\pi x}{A} \sin\frac{n\pi y}{B};$$

$$\psi_x^i(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{3+imn}(t) \cos\frac{m\pi x}{A} \sin\frac{n\pi y}{B};$$

$$\psi_y^i(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{3+I+imn}(t) \sin\frac{m\pi x}{A} \cos\frac{n\pi y}{B};$$

$$P = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn}(t) \sin\frac{m\pi x}{A} \sin\frac{n\pi y}{B};$$

$$i = \overline{1, I},\quad m = \overline{1, M},\quad n = \overline{1, N}. \tag{8}$$

Hence, the problem of multilayer plate vibrations for each of the values m and n

reduces to integration of a system consisting of one ordinary differential equation and $2I+2$ algebraic equations.

The algebraic equations express $\Phi_{1mn}(t)$, $\Phi_{2mn}(t)$, $\Phi_{3+imn}(t)$, $\Phi_{3+I+imn}(t)$ to $\Phi_{3mn}(t)$:

$$\Phi_{1mn}(t) = \xi_1 \Phi_{3mn}(t),$$

$$\Phi_{2mn}(t) = \xi_2 \Phi_{3mn}(t),$$

$$\Phi_{3+imn}(t) = \xi_{3+i} \Phi_{3mn}(t),$$

$$\Phi_{3+I+imn}(t) = \xi_{3+I+i} \Phi_{3mn}(t), \quad i = \overline{1, I}, \tag{9}$$

resulting in the problem of non-stationary strain in a multilayer plate reducing to integration of an ordinary differential equation,

$$C_\rho^I \Phi_{3mn,tt} + L\,\Phi_{3mn} = q_{mn}, \tag{10}$$

where

$$L = C_3^I\left(\frac{m^2\pi^2}{A^2} + \frac{n^2\pi^2}{B^2}\right) + \frac{m\pi}{A}\sum_{i=1}^{I} \alpha_3^i \xi_{3+i} + \frac{m\pi}{B}\sum_{i=1}^{I} \alpha_3^i \xi_{3+I+i}, \quad i = \overline{1, I}.$$

At this, the following initial conditions are to be taken into account:

$$\Phi_{3mn,t}(0) = \Phi_{3mn}(0) = 0.$$

Using the Laplace integral transform (Kokhmaniuk *et al.*, 1989), the solution of eqn (10) can be presented in the form

$$\Phi_{3mn}(t) = \Phi_{3mn}(t_0)\cos\omega(t-t_0) + \frac{\Phi_{3mn}(t_0)_{,t}}{\omega}\sin\omega(t-t_0)$$
$$+ \frac{1}{C_\rho^I\omega}\int_{t_0}^{t} q_{mn}(\tau)\sin\omega(t-\tau)\,\mathrm{d}\tau, \tag{11}$$

where

$$\omega^2 = L/C_\rho^I.$$

The integration interval is divided into sections of length Δt, i.e. $t = s\Delta t$. At a small Δt interval, the function $q_{mn}(\tau)$ may be assumed to be constant, equal to its average value $q_{mn}(\tau) = q_{mn}^s$, and factored outside the integral sign. Thus, after finding the integral, solution (II) is transformed to the form

$$\Phi_{3mn}^{s+1} = \Phi_{3mn}^s C + \frac{\Phi_{3mn,t}^s}{\omega} S + \frac{q_{mn}^s}{C_\rho^I\omega^2}(1-C),$$

$$\Phi_{3mn,t}^{s+1} = -\Phi_{3mn}^s \omega\, S + \Phi_{3mn,t}^s C + \frac{q_{mn}^s}{C_\rho^I\omega} S.$$

Here $C = \cos\omega\Delta t$, $S = \sin\omega\Delta t$.

The remaining coefficients of expanding the displacements into Fourier series are found from eqn (9). The displacements are obtained by summation of series (8) with account of expression (1), and the stresses are calculated according to Hooke's law, eqn (3).

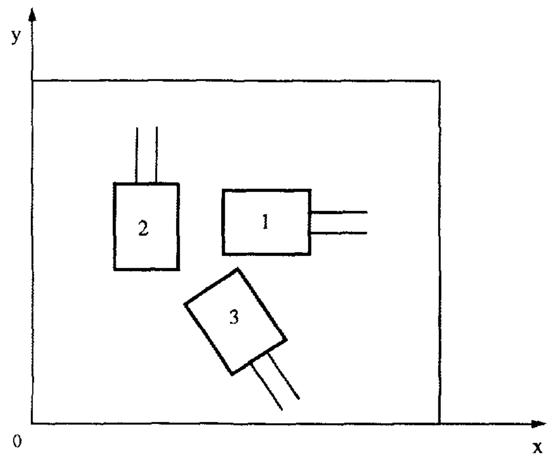

Fig. 2. Strain gauges allocation scheme.

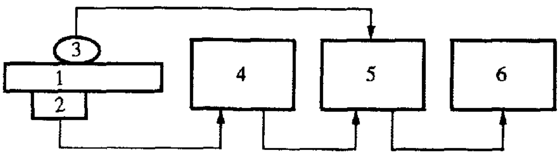

Fig. 3. Block diagram of experimental system.

EXPERIMENTAL METHOD OF INVESTIGATION

The technique of testing plates and measuring strains under impact loading provides detection of the current time-dependent strain values and measurement of time intervals. These requirements are met by the appropriate selection of instrumentation, strain gauges, bonding technique, as well as the systems of loading and calibration of the amplification channel.

The most convenient technique of strain measurement under impact action is the dynamic wide-range strain measurement. In this work, small-base (measurement base 1 mm) foil resistance strain gauges were used.

Securing the plate over its periphery models the conditions of free support.

Loading is performed by letting a steel ball indentor drop on the plate. A three-component rosette of strain ganges (Fig. 2) is bonded to the side opposite to that of the load application.

The signals from the strain gauges (2) (Fig. 3), bonded to plate (1), are fed to the strain gauge amplifier (4), and then to the instrumentation and computing complex. The triggering sensor (3) is placed on the side of the plate to which the load is applied, and it is located 3 cm away from the point of load application. The sensor is a piezoceramic one, which responds to a displacement perpendicular to the plane of its securing. Upon application of an impact load to the plate, the sensor outputs a signal to the syncropulse generator (SPG). It is used for simultaneous triggering of all measurement channels upon receiving a signal from the triggering sensor. The generator is made in the CAMAC standard. Also, the CAMAC crate (5) accomodates a clock-pulse generator (CPG), analogue-to-digital converters (ACD), with the sampling frequency 40 MHz and a 1K memory, as well as a crate controller. The CPG is intended for clocking all the measurement channels. An adapter located in the computer (6) and the crate controller serve to maintain a dialogue between the computer and the units arranged in the CAMAC crate. The CPG, SPG and ADC modes of operation can be set on the computer keyboard.

The strain gauge amplifier was designed at the Institute for Problems in Machinery of the Ukrainian Academy of Sciences (Kharkov). It is an eight-channel wide-range amplifier operating on the principle of amplitude modulation of voltage by a 1 MHz carrier frequency feeding the measuring bridge.

The strain is measured by a bridge circuit. One bridge arm is used in the calibration circuit, and the remaining two arms are employed in the strain gauge amplifier. To minimize the current in the measurement diagonal, an adjustment of the active and reactive resistance components is performed. Prior to testing, after balancing the amplification channel, it is calibrated to set the dependence $\varepsilon = \varepsilon(U)$, where U is the ADC output voltage.

Strain gauge amplifier specifications:

number of measurement channels	8
carrier frequency, kHz	1000
operating frequency range, kHz	0.04–200
amplitude–frequency response non-linearity, dB, max	± 1.2
minimal detected strain, relative strain units (RSU)	30×10^{-6}
dynamic range, dB	80
calibration range, RSU	30×10^{-6}–2.4×10^{-3}
length of transmission lines, m, max	20
resistance of strain ganges, Ω	50–200.

The system is controlled by the IBM PC/AT computer. The application program package allows the system to perform the following work:

create a file of the test sequence;
test the ADC and certify its operability;
carry out the experiment with recording of data in the working file;
carry out proximate analysis;
graphic presentation with subsequent output to the monitor or printer;
search of the maximum and minimum values;
carry out spectral analysis.

NUMERIC AND EXPERIMENTAL RESULTS

Plate vibrations induced by impact loading were considered. Loading was effected by dropping a steel ball with the mass M from the height H in the centre of the plate ($x_0 = A/2, y_0 = B/2, z_0 = 0$). The area of interaction of the indentor and plate is a round spot of radius a. The radius of the loading spot a and coefficient k [see eqn (7)] are determined as follows (Dinnik, 1952):

$$a = \left(\frac{3}{16} F R(\theta_1 + \theta_2)\right)^{1/3},$$

$$k = \left(\frac{9}{256} \frac{(\theta_1 + \theta_2)^2}{R}\right)^{1/3},$$

$$\theta_1 = \frac{4(1 - \nu_1^2)}{E_1}, \theta_2 = \frac{4(1 - \nu_2^2)}{E_2}.$$

Here E_1, ν_1 are the plate mechanical characteristics; E_2, ν_2 are the ball characteristics; R is the ball radius.

It is assumed that the ball contact pressure is distributed over the loading spot by the law

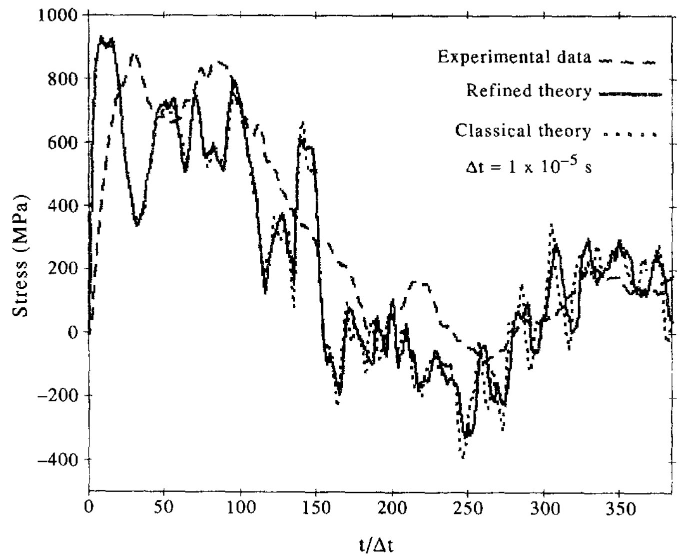

Fig. 4. Impact of a steel ball on a steel plate.

$$P = P_0(t)\left[1-\frac{(x-x_0)^2+(y-y_0)^2}{a^2}\right]^{1/2},$$

and the resultant in eqns (6) and (7) is equal to

$$F = \frac{2}{3}\pi a^2 P_0.$$

Here x_0, y_0 are the coordinates of the ball and plate contact point at the initial moment of time. The coefficients of expanding the load into a series are found from

$$q_{mn} = \frac{12F}{AB\rho_{mn}^2}\sin\frac{m\pi x_0}{A}\sin\frac{n\pi y_0}{B}\left(\frac{\sin\rho_{mn}}{\rho_{mn}}-\cos\rho_{mn}\right),$$

where

$$\rho_{mn} = \pi\left[m^2\left(\frac{a}{A}\right)^2+n^2\left(\frac{a}{B}\right)^2\right]^{1/2}.$$

A comparison of the experimental and calculation results was carried out for a one-layer steel rectangular plate having the dimensions $A = 0.675$ m, $B = 0.18$ m and thickness $h = 0.005$ m. The mechanical characteristics of the plate and ball characteristics are $E = 2.1\times 10^5$ MPa, $\nu = 0.3$, $\rho = 7.85\times 10^3$ kg m^{-3}, ball radius $R = 0.03$ m, height of dropping $H = 2.3$ m.

The strain was measured at the central point of the surface of a plate not subjected to the action of a load ($x = 0.3375$ m, $y = 0.09$ m, $z = 0.005$ m). The stresses corresponding to these deformations were determined by Hooke's law:

$$\sigma_x = \frac{E}{2(1-\nu^2)}\left[(1+\nu)(\varepsilon_1+\varepsilon_3)+(1-\nu)\left((\varepsilon_1-\varepsilon_3)^2+(\varepsilon_1-2\varepsilon_2+\varepsilon_3)^2\right)^{1/2}\right]$$

$$\sigma_y = \frac{E}{2(1-\nu^2)}\left[(1+\nu)(\varepsilon_1+\varepsilon_3)-(1-\nu)\left((\varepsilon_1-\varepsilon_3)^2+(\varepsilon_1-2\varepsilon_2+\varepsilon_3)^2\right)^{1/2}\right].$$

Here ε_1, ε_2, ε_3 are the relative elongations measured by means of a rosette comprising three strain gauges (Fig. 2).

Figure 4 shows the calculated and experimental time dependences of σ_y.

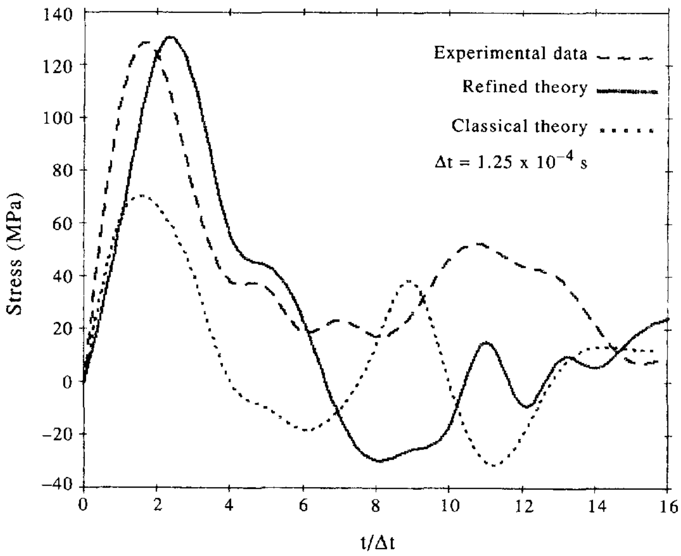

Fig. 5. Impact of a steel ball on a multilayer plate.

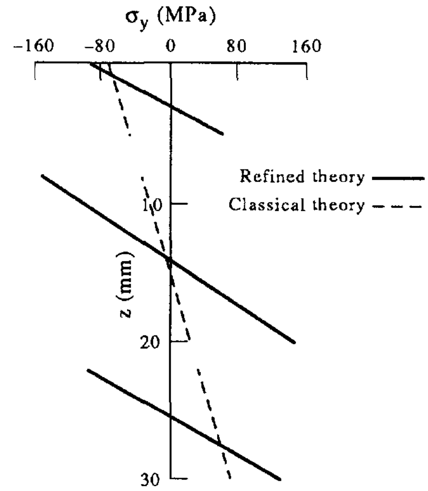

Fig. 6. Variation of stresses σ_y through the thickness of a multilayer plate.

A similar investigation was carried out for a five-layer plate. In this case, the maximum spot radius a and coefficient k were determined on the basis of experimental data. They are: $a = 0.0095$ m, $k = 1.32 \times 10^{-4}$. The dimensions of the plate were $A = 0.82$ m, $B = 0.24$ m; the composition of the layers was $h_1 = 0.005$ m, $h_2 = 0.003$ m, $h_3 = 0.012$ m, $h_4 = 0.002$ m, $h_5 = 0.008$ m. The carrying layers ($i = 1, 3, 5$) were made of silica glass ($E = 6.8 \times 10^4$ MPa, $\nu = 0.22$, $\rho = 2.5 \times 10^3$ kg m^{-3}) and interconnected by layers ($i = 2, 4$) of polymer material ($E = 2.8 \times 10^2$ MPa, $\nu = 0.39$, $\rho = 1.2 \times 10^3$ kg m^{-3}). The load was applied to the external surface of the first layer ($x_0 = 0.41$ m, $y_0 = 0.12$ m, $z_0 = 0$), and the stresses σ_y^5 were calculated in the centre of the plate on the external side of the fifth layer ($x = 0.41$ m, $y = 0.12$ m, $z = 0.03$ m) (see Fig. 5).

For comparison, the numerical results received by using a classical multilayer shells theory (Ambartsumyan, 1961), based on the non-deformable normal hypothesis for a package, are presented in Figs 4 and 5.

Figure 6 shows the variation of stresses σ_y through the thickness of the above-mentioned five-layer plate in a time moment $t = 3.75 \times 10^{-4}$ s, when they reach the maximum.

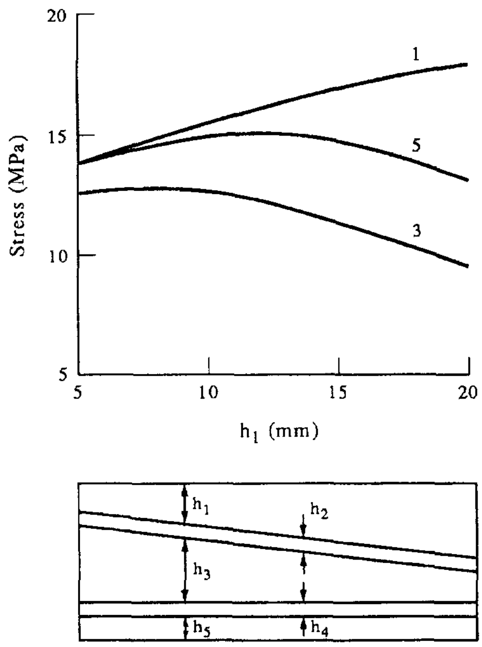

Fig. 7. Dependence of stresses on the composition of the layers package.

For a homogeneous plate the satisfactory agreement of results obtained on the basis of classical and refined theories is to be observed. When elastic layers properties differ significantly the classical theory gives a considerable error, which is confirmed by Reddy's works (1989, 1993).

Non-stationary vibrations of a square-shaped multilayer plate induced by a uniformly distributed load $P = P_0 H(t)$, where $H(t)$ is Heaviside function, were also considered. The coefficient of expanding the load P into a Fourier series has the form

$$q_{mn} = \frac{4P_0}{mn\pi^2}(1-(-1)^m)(1-(-1)^n).$$

The plate dimensions were $A = 0.5$ m, $B = 0.5$ m, the load intensity was $P_0 = 0.1$ MPa. Layers 1, 3, 5 were made of silica glass, and layers 2, 4 are a polymer material.

The influence of the relationship of the thicknesses of the carrying layers on the modules of the maximum values of the stresses tensor normal component in the plate layers was investigated for $h_1 + h_3 = 0.025$ m, $h_2 = h_4 = 0.003$ m, $h_5 = 0.005$ m. The upper part of Fig. 7 shows the absolute values of the maximum stresses $|\sigma^i_{max}|$, depending on the value of h_1. The stresses in the bond layers are not shown, since their presentation in the figure in the accepted scale is difficult. The nomogram in the lower part of the figure allows determination of the relationship of the carrying layers in the package by the value of h_1.

CONCLUSION

The main results of the work can be stated as follows:

(1) Dynamic equation of a linear refined theory of multilayer plates has been derived, and the method of investigating non-stationary vibrations of plates under impulse and impact loading has been developed on this basis.
(2) A technique and a system of instrumentation for carrying out dynamic stress measurements under impact and impulse loading of structures has been offered and developed.

(3) Experimental and theoretical results obtained under impact loading have been compared.
(4) The influence of structural parameters on the stress–strained state of multilayer plates under impulse loading has been investigated. It has been shown that at constant package thickness the composition may be selected so that the layers optimally take up the load being applied.

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